The Field Equations of Field Topology Theory: A Specification

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Author's Note on Methodology and Status

This paper was developed through an iterative process involving collaborative AI systems (Claude for theoretical development and synthesis, Gemini for adversarial review and literature search) and the author's geometric intuition. This methodology is disclosed because intellectual honesty about process is as important as intellectual honesty about results.

The paper contains three distinct categories of claim, and the reader should know which is which.

Established results cited from the literature: the Finkelstein-Rubinstein theorem, the Kossowski-Kriele junction conditions, the McKay correspondence between 2T and E_6, the Koide formula's six-decimal-place agreement with experiment, the Makaryev-Shcherb derivation of $b = -\sqrt{2}$ from Willmore energy, the Tegmark analysis of signature viability, the Alexandre-Gielen-Magueijo result connecting signature change to cosmological constant shifts, and the Chen-Kantowski proof regarding electromagnetic attenuation. These are peer-reviewed or well-established mathematical results. They are not in dispute.

Requirements derived from established results: the nine requirements specified in this paper follow from combining established results with the FTT framework. They are logical consequences of the programme's premises. Each requirement is individually falsifiable, and the falsification conditions are stated explicitly. These requirements have survived adversarial review in the sense that no mathematical impossibility has been identified in any individual requirement — though the combination of all nine simultaneously remains unproven.

Geometric motivations and intuitive arguments: several passages in this paper describe physical pictures that motivated the requirements but do not constitute derivations. These include: the rotating sphere picture for charge and confinement (Section 10), the Pascal triangle cascade for baryogenesis (Section 11), and the ocean/ice metaphor for the Lorentzian/Euclidean phase transition. These are explicitly flagged as motivations throughout the text. They suggest what the mathematics should produce but they do not produce it. The reader should not mistake geometric motivation for mathematical proof.

Five specific claims in this paper were subjected to adversarial review and revised in response. The revisions are documented in the relevant sections. Claims that were found to be overstated have been demoted from derivations to motivations. Claims that faced fatal objections have been either killed or constrained with explicit conditions under which they survive. In each case, the nature of the objection and the revision are stated rather than hidden.

The author's background is in clinical ophthalmology, not theoretical physics. The ideas in this paper arise from geometric pattern recognition rather than formal mathematical training. This is both a strength (the geometric intuitions have produced results that formal approaches have not) and a limitation (the formalisation of those intuitions into rigorous mathematics remains incomplete and may require collaboration with mathematical physicists). The paper is published as a specification — a statement of what the mathematics must do — precisely because the author cannot yet write the mathematics that does it.

No AI system is listed as a co-author. AI tools were used as computational and analytical instruments, not as intellectual originators. The ideas, errors, and revisions are the author's responsibility.

Abstract

Field Topology Theory proposes that particles, forces, and quantum numbers emerge as topological configurations of a single continuous spacetime field. This paper does not derive the FTT field equations — it specifies the requirements any such equation must satisfy. Nine requirements are identified and mapped to existing mathematical structures: the equation must reduce to general relativity far from defects; admit stable topological defects with $\mathbb{Z}_2$ winding; remain well-defined through metric signature transitions; support Euclidean phase solutions corresponding to dark matter; produce a minimum action scale from vacuum topology; generate discrete mass families matching observed ratios via binary tetrahedral symmetry; admit fractional topological twist stable only in confined bound states; exhibit topological path dependence in defect nucleation; and yield stable defect solutions exclusively in 3+1 signature. A constructive survey of the mathematical physics literature confirms that individual mechanisms satisfying each requirement exist — spanning Skyrme models, scalar-tensor theories, topological soliton physics, and braid pre-geometries — but no single equation currently combines them. The precise mathematical gaps preventing unification are identified. The specification is offered as a roadmap for construction of the complete field equation.

Section 1: Introduction — The Ocean and the Equation

The spacetime field is an ocean. Matter is persistent patterns — whirlpools, ripples, standing waves, frozen regions. All patterns emerge from one medium governed by one equation. This paper specifies what that equation must do.

The Standard Model describes particles, forces, and quantum numbers with extraordinary precision but treats them as inputs — 19 free parameters measured, not derived. General relativity describes gravity as spacetime curvature but says nothing about what curves spacetime at the quantum scale. The two theories are formulated on incompatible foundations: quantum field theory on a fixed background, general relativity as a dynamical background with no quantum structure.

Field Topology Theory proposes that both frameworks are limiting cases of a single field equation governing a single spacetime field. Particles are stable topological defects in this field. Forces are gradient interactions between defects. Quantum numbers are topological invariants. Dark matter is a phase transition in the metric signature. The programme's prior results — the Finkelstein-Rubinstein derivation of fermion statistics from Lorentzian topology, the Koide mass formula from binary tetrahedral symmetry, the clock-rate formulation of gravitational effects, the geometric selection model for fast radio bursts — are published separately. This paper does not derive the field equation. It specifies the nine requirements any candidate equation must satisfy, maps each requirement to existing mathematical structures, identifies the precise gaps, and offers the specification as a roadmap.

The paper is structured as follows. Section 2 presents the nine requirements. Sections 3 through 11 treat each requirement individually: stating the physical motivation, identifying the closest existing mathematical structure, and specifying what is missing. Section 12 addresses the design philosophy — why discrete and continuous descriptions must emerge from one equation at different scales. Section 13 consolidates the gaps into a research programme.

Section 2: The Nine Requirements

Any candidate FTT field equation must satisfy all nine requirements simultaneously. They are not independent — several are mutually constraining, and the design philosophy (Section 12) imposes a further meta-constraint. They are presented here in logical order, beginning with the most fundamental.

Requirement 0 — Signature Selection. The equation must admit solutions in arbitrary metric signature but yield stable, finite-energy topological defects only in 3+1 (Lorentzian) signature. This is not an input — it must emerge from the structure of the equation itself.

Requirement 1 — General Relativistic Limit. In the Lorentzian regime, far from topological defects, the equation must reduce to Einstein's field equations.

Requirement 2 — Topological Defects with $\mathbb{Z}_2$ Winding. The equation must admit stable, finite-energy solutions carrying a $\mathbb{Z}_2$ topological charge. By the Finkelstein-Rubinstein theorem, such defects automatically carry half-integer spin and fermionic exchange statistics.

Requirement 3 — Signature Transition. The equation must remain well-defined at $\det(g) = 0$ — the surface where the metric signature transitions between Lorentzian and Euclidean.

Requirement 4 — Euclidean Phase Solutions. Beyond the signature transition surface, the equation must admit Euclidean metric solutions corresponding to dark matter.

Requirement 5 — Quantum of Action from Vacuum Topology. The equation must produce a natural minimum action scale $\hbar$ emerging from the topological structure of the vacuum itself.

Requirement 6 — Mass Hierarchy from Binary Tetrahedral Symmetry. Defect solutions must come in discrete families whose mass ratios match the Koide formula with parameter $b = -\sqrt{2}$, anchored to the binary tetrahedral group 2T via the McKay correspondence to E_6.

Requirement 7 — Confinement. The equation must admit solutions where the topological twist decomposes into fractional values stable only in bound combinations summing to a complete rotation.

Requirement 8 — Baryogenesis. The equation must exhibit topological path dependence in defect nucleation, producing a slight chirality excess corresponding to the observed baryon asymmetry.

These nine requirements are mutually constraining. No requirement can be satisfied by a mechanism that violates another.

Section 3: Requirement 0 — Signature Selection

3.1 Physical Motivation

The universe has three spatial dimensions and one temporal dimension. The FTT field equation must do better than taking this as given: 3+1 must emerge as the only signature in which the equation's topological defect solutions are stable.

The argument does not rest on classical fluid dynamics. It is topological, not hydrodynamic. Earlier formulations of this argument invoked Helmholtz's vortex theorems. This framing is abandoned. Helmholtz's theorems apply to inviscid fluids under restrictive conditions that do not hold for a metric field theory. The dimensionality argument rests on homotopy theory and the topological classification of defects.

The configuration space of metrics on a d-dimensional spatial manifold has dimension-dependent homotopy groups. In three spatial dimensions, the configuration space of Lorentzian metrics has $\pi_1(RP^3) = \mathbb{Z}_2$ — a non-trivial fundamental group that permits stable topological defects carrying fermionic statistics via the Finkelstein-Rubinstein theorem. This result is specific to SO(3), the rotation group in three dimensions. In other spatial dimensions, the relevant rotation group is SO(d), whose covering space and fundamental group differ. The $\mathbb{Z}_2$ structure that gives fermion statistics is not guaranteed in other dimensions.

In four or more spatial dimensions, an additional topological obstruction applies: 1D knotted structures can be continuously unknotted. This is an established result in geometric topology — not dependent on any fluid analogy. Topological defects defined by non-contractible loops lose their topological protection in higher spatial dimensions.

A timelike dimension with distinct metric character is required for clock-rate gradients to exist between different points on a topological configuration. Without such gradients, the configuration has no internal dynamics and no mechanism for stabilisation. Zero timelike dimensions give static structures with no tension. More than one timelike dimension produces ultrahyperbolic PDEs whose initial value problems are ill-posed.

The "Why Discrete?" paper (doi:10.5281/zenodo.20071696) establishes the homotopy-theoretic chain in full: $\pi_1(RP^3) = \mathbb{Z}_2 \rightarrow$ Finkelstein-Rubinstein $\rightarrow$ fermion statistics $\rightarrow$ asymptotic framing resolves diffeomorphism invariance.

3.2 Existing Mathematical Structures

Tegmark (1997). Demonstrated that in signatures other than 3+1, either the wave equation is ill-posed or stable structures cannot exist. The FTT argument is deeper: it asks where topological structure can exist at all. Tegmark's result is a consequence, not the foundation.

Topological classification of defects by spatial dimension. The homotopy group classification is dimension-dependent. The specific result $\pi_1(RP^3) = \mathbb{Z}_2$ is a property of SO(3). In other dimensions, the relevant fundamental groups differ.

Skyrme model dimensionality dependence. Skyrmion stability relies on the Derrick scaling argument being evaded by the quartic term. In other dimensions, the balance changes and stable solitons may not exist.

3.3 The Gap

No existing work proves that stable topological defects in a metric field theory exist exclusively in 3+1 signature. The components are present. The specific calculation — computing the defect spectrum as a function of signature and showing stability only in (1,3) — cannot be performed until the field equation is written. Requirement 0 functions as a consistency check on the completed equation.

3.4 Falsifiability

Falsifiable within the theoretical programme: if the completed equation admits stable defects in other signatures, the equation fails.

Section 4: Requirement 1 — General Relativistic Limit

4.1 Physical Motivation

General relativity is not wrong. Any field equation extending it must reproduce every confirmed prediction in the appropriate limit — the Lorentzian bulk, far from defects, at macroscopic scales.

4.2 Existing Mathematical Structures

Scalar-tensor theories demonstrate that the GR limit can be achieved by construction.

The Skyrme model demonstrates exponential localisation of non-linear terms.

Effective field theory explains why corrections are suppressed by powers of the defect scale.

GW170817 is satisfied structurally: one ocean, one speed for both electromagnetic and gravitational waves.

4.3 The Gap

No gap in principle. The construction strategy — $S = S_{EH} + S_{top} + S_{phase}$ with additional terms negligible in the bulk — is standard.

4.4 Falsifiability

PPN parameters must match GR within observational bounds. Deviations near phase boundaries are predictions, not violations.

Section 5: Requirement 2 — Topological Defects with $\mathbb{Z}_2$ Winding

5.1 Physical Motivation

Particles are stable, localised, fermionic. FTT requires all of this to emerge from the field equation via $\pi_1(RP^3) = \mathbb{Z}_2$ and the Finkelstein-Rubinstein theorem. Fermions are what you get when a Lorentzian metric field has a knot in it.

5.2 Existing Mathematical Structures

Finkelstein and Rubinstein (1968): the foundational result. $\mathbb{Z}_2$ solitons are quantised as fermions.

The Skyrme model: proof of concept for particles from topology.

Asymptotic framing: resolves diffeomorphism invariance at the boundary.

Derrick's theorem: must be evaded by a higher-order stabilising term.

5.3 The Gap

The gravitational Wess-Zumino-Witten term — the single most important missing piece. Must be constructed in the space of spacetime metrics. No such term exists in the literature. The η-invariant of Atiyah-Patodi-Singer and the Dai-Freed theorem suggest possible directions.

5.4 Falsifiability

Any confirmed violation of the Pauli exclusion principle (VIP experiment, Gran Sasso) would falsify the Finkelstein-Rubinstein mechanism. The exclusion principle in FTT is a topological theorem. It cannot be slightly violated.

Section 6: Requirement 3 — Signature Transition

6.1 Physical Motivation

FTT proposes that regions exist where the metric signature has transitioned to Euclidean (+,+,+,+). The transition surface — where det(g) = 0 — is where the domain of general relativity ends. The FTT field equation must remain well-defined through this surface. This is what makes FTT genuinely different from modified gravity theories.

6.2 Existing Mathematical Structures

Kossowski and Kriele (1993). Established that for smooth signature change, the extrinsic curvature must vanish identically on the transition surface (K_ij = 0).

Alexandre, Gielen, and Magueijo (JCAP 2024). Showed that signature change implies a shift in the effective cosmological constant — the ISW anchor.

Chen and Kantowski (2009). Proved that a real modified g_{00} keeps the wave equation hyperbolic and produces zero electromagnetic attenuation.

Hartle-Hawking no-boundary proposal. FTT treats signature transition as a local phenomenon occurring wherever the field finds it energetically favourable, not just at the beginning of time.

6.3 The Gap

The gap has three parts.

First: the dynamics of the transition. The equation must contain a phase transition term with minima at both Lorentzian and Euclidean signatures.

Second: regularity at det(g) = 0. The equation must be formulated in variables that remain finite, likely requiring densitised or connection variables.

Third: the surface energy question and the pressure-balance resolution. Under the smooth Kossowski-Kriele junction condition K_ij = 0, the Lanczos thin-shell formalism yields exactly zero surface energy density at the transition surface. FTT does not contest this result — it agrees with it.

However, a subtlety arises. Under the Israel junction conditions, any discontinuous jump in bulk pressure across a boundary generates a non-zero jump in the extrinsic curvature — which would violate the Kossowski-Kriele smoothness condition. A sharp pressure discontinuity and K_ij = 0 are mathematically incompatible.

The resolution is that the phase boundary is not a mathematical surface. It is a finite-thickness transition region across which the metric character changes continuously. The Lorentzian phase does not meet the Euclidean phase at a sharp edge. The metric signature transitions over a characteristic length scale — the decay length over which oscillation dies out as the timelike dimension loses its distinct character. This decay length is set by the minimum action unit $\hbar$: each oscillation cycle costs $\hbar$, and as the available energy for oscillation decreases through the transition region, the oscillation amplitude falls to zero over a finite distance.

Within this transition region, the pressure balance is not a discontinuity but a gradient. The dynamic pressure of the oscillating Lorentzian field decreases continuously. The effective pressure of the static Euclidean phase remains constant. The point where the decreasing dynamic pressure equals the Euclidean phase pressure is a location within the continuous transition — analogous to how the heliopause is not a sharp wall but a region where solar wind pressure falls below interstellar medium pressure.

The Kossowski-Kriele conditions apply asymptotically on either side of the transition region: the metric is smoothly Lorentzian far inside, smoothly Euclidean far outside, and the degenerate surface $\det(g) = 0$ is approached continuously from both sides. The extrinsic curvature vanishes asymptotically, satisfying the junction conditions in the distributional sense without requiring a sharp discontinuity at any single surface.

The minimum size of a stable Euclidean region is then set by the geometry of the transition region. A region too small to contain a full transition — where the decay length exceeds the radius — cannot sustain a stable interior. The minimum radius is of order the decay length, which is set by $\hbar$ and the local energy density.

6.4 Falsifiability

The clock-rate paper demonstrates differential expansion matching Seifert et al. Pantheon+ results with $\ln B > 5$. The ISW paper demonstrates enhanced negative ISW consistent with the Hansen et al. hot void anomaly. Chen-Kantowski 2009 confirms zero electromagnetic attenuation. GW170817 is automatically satisfied. The Roman Space Telescope (launch 2027) will provide the discriminating observation: FTT predicts differential expansion localised to void boundaries, not uniform acceleration.

Section 7: Requirement 4 — Euclidean Phase Solutions

7.1 Physical Motivation

Dark matter is the most successful failure in physics. Its gravitational effects are observed everywhere. No particle has ever been detected. The absence of detection is data.

FTT's identification: dark matter is not a particle. It is ice — regions of spacetime where the metric signature has transitioned from Lorentzian to Euclidean. These frozen regions gravitate but do not interact electromagnetically in the standard sense.

7.2 Existing Mathematical Structures

Euclidean quantum gravity and gravitational instantons. Well-developed mathematics that FTT reinterprets as descriptions of real regions.

NFW profile. Any Euclidean phase solution must reproduce observationally equivalent lensing and rotation curves.

MOND phenomenology. The acceleration scale a_0 may emerge naturally from the phase transition energetics.

The splashback radius. Recent work defines the physical edge of dark matter haloes as a kinematic phase boundary rather than a gravitational potential boundary. The offset between the collisionless splashback radius and the baryonic shock radius (factor 1.3 to 2.0, predominantly toward voids) is consistent with FTT's identification of both boundaries as aspects of the Lorentzian/Euclidean phase transition.

7.3 The Gap

The explicit construction of the Euclidean phase solution and its gravitational effect. The interior geometry, gravitational mass, and MOND connection must all be computed from the field equation.

Dark matter substructure and the environment-dependent cutoff. Particle dark matter models predict a universal low-mass cutoff determined by the particle's free-streaming length. Euclidean phase dark matter predicts an environment-dependent cutoff: the minimum Euclidean droplet size is set by the local energy density through the transition region decay length (Section 6.3). In dense environments, the decay length is short but the dynamic pressure is high, requiring larger droplets to sustain pressure balance. In sparse environments, smaller droplets can stabilise. The low-mass cutoff of the halo mass function should therefore vary with local cosmic web environment: higher in clusters, lower in voids.

This is consistent with high-resolution simulations demonstrating that the halo mass function is profoundly environment-dependent. The 2024 Homma et al. result — over 500 satellite galaxies extrapolated from the Subaru HSC survey — is naturally accommodated: the satellites exist because the local environment permitted small Euclidean droplets to stabilise.

The shape of the low-mass cutoff is the discriminating test. Particle dark matter (WDM, FDM) predicts a universal sharp cutoff independent of environment. FTT predicts an environment-dependent cutoff. Future surveys resolving the faintest satellites across different cosmic web environments will distinguish between these predictions.

Mathematical challenge. The Cauchy initial value problem for field equations at a Lorentzian/Euclidean interface is not well-posed under standard formulations. High-frequency modes at the interface can grow without bound, potentially destroying mathematical predictability. This is a genuine obstruction connecting directly to Requirement 3. The resolution may lie in the finite-thickness transition region: if the metric signature never reaches $\det(g) = 0$ but only approaches it asymptotically, the strict Cauchy problem at a degenerate surface is avoided. The field equation would govern a continuous transition, not a singular boundary. If no well-posed formulation exists, the Euclidean phase identification fails regardless of its observational consistency.

7.4 Falsifiability

Direct detection experiments will never find a dark matter particle. This is absolute. A confirmed detection falsifies FTT's dark matter identification immediately. The environment-dependent halo mass function cutoff provides a discriminating prediction: if the cutoff is universal, the pressure-balance model is falsified. The neutron star mass limit of approximately 2.298 solar masses provides a specific numerical prediction.

Section 8: Requirement 5 — Quantum of Action from Vacuum Topology

8.1 Physical Motivation

$\hbar$ is the minimal symplectic flux of the 4D Lorentzian vacuum. The electron is the minimum stable defect costing one $\hbar$ unit. The vacuum defines $\hbar$, and the electron inherits it.

8.2 Existing Mathematical Structures

Geometric quantisation: the integrality condition of the symplectic form provides the rigorous pathway.

Loop quantum gravity: demonstrates quantisation from gravitational configuration space topology.

Chern-Simons theory: the cleanest example of quantisation from topology.

8.3 The Gap

The explicit computation of the minimal symplectic flux using the ADM symplectic form evaluated on the $\mathbb{Z}_2$ generator of $\pi_1(RP^3)$. This can be attempted now, without the full field equation.

8.4 Falsifiability

Any observed violation of exact quantisation — a non-integer multiple of $\hbar$ for any action integral — would falsify the topological identification.

Section 9: Requirement 6 — Mass Hierarchy from Binary Tetrahedral Symmetry

9.1 Physical Motivation

The Koide formula relates the three charged lepton masses to six decimal places. The parameterisation reveals Z_3 symmetry from 2T and $b = -\sqrt{2}$ from the Clifford torus Willmore energy. The three fermion generations are the three ways a topological defect can sit in the binary tetrahedral symmetry of the vacuum.

9.2 Existing Mathematical Structures

McKay correspondence: 2T maps exactly to E_6.

Makaryev and Shcherb (arXiv:2602.0035): independent derivation of $A = \sqrt{2}$ from Willmore energy.

Sumino (2009): U(3) family gauge symmetry as RG protection.

9.3 The Gap

Three gaps: the dynamical mechanism for $b = -\sqrt{2}$; the Sumino protection from 2T automorphisms; extension beyond charged leptons. The Koide angle $\delta = 2/9$ remains underived.

9.4 Falsifiability

The Koide formula predicts the tau mass to be 1776.969 MeV. Current value: 1776.86 ± 0.12 MeV. Any future measurement outside this prediction falsifies the mechanism.

Section 10: Requirement 7 — Confinement

10.1 Physical Motivation

Quarks are never observed alone. FTT's explanation: the full topological twist is a lepton. Quarks are fractional twists — incomplete configurations that are meaningful only relative to the complete rotation they are a fraction of. Confinement is geometric impossibility: a fraction cannot exist without the whole.

10.2 Geometric Motivation and Its Limits

The rotating sphere picture provides geometric motivation for why charge might relate to the twist character of topological defects and why fractional charges might correspond to incomplete topological configurations. Gravity appears as a radial clock-rate gradient with no rotational character. Charge appears as a gradient with twist. Fractional charge appears as an incomplete twist requiring completion.

However, this picture is not a derivation. A latitude on a continuous sphere is a coordinate, not a topological invariant. Quantised charge requires discrete topological protection — it must map to a homotopy class, not to a continuous parameter. The continuous sphere geometry suggests what the answer should look like. The discrete topology of the defect configuration space must deliver it.

The claim that the closure constraint — three fractional twists summing to a complete rotation — geometrically motivates SU(3) remains as stated: a motivation, not a derivation. The derivation requires computing the symmetry group of the constrained configuration space of three fractional topological charges on S^3 subject to the closure condition (Section 13.3, Calculation 2). Until that calculation is completed, the SU(3) identification is a target, not a result.

10.3 Existing Mathematical Structures

Bilson-Thompson's braid model: first-generation fermions as braids with 1/3 twist. The direct predecessor.

Skyrmion baryon number: topological confinement of third-integer charges demonstrated in a continuous field theory.

10.4 The Gap

The central gap is deriving SU(3) from the closure constraint — a group-theoretic calculation that can be attempted now. The connection between the Z_3 of colour and the Z_3 of generations is deeply suggestive but unproven.

10.5 Falsifiability

Falsifiable by the detection of a free quark. The quantitative test is the string tension, which must match approximately $(440\ \text{MeV})^2$.

Section 11: Requirement 8 — Baryogenesis

11.1 Physical Motivation

The universe contains matter. The Standard Model's CP violation is too small by orders of magnitude. FTT proposes a topological cascade mechanism: the first defect selects a chirality, biasing subsequent formation.

11.2 The Topological Cascade

CP violation in this framework is path dependence in a topological cascade, not a Lagrangian parameter. The three Sakharov conditions are satisfied automatically through topology: baryon number violation through defect formation; CP violation through path dependence; departure from equilibrium through irreversible topological commitment.

Critical constraint: the domain wall exclusion. If the chirality selection is stochastic and local, causally disconnected regions of the early universe would independently select different chiralities. This would produce macroscopic domains of matter and antimatter separated by domain walls — a configuration decisively excluded by observation.

The cascade mechanism survives this objection only if the first symmetry break occurs when the entire observable universe is within a single causal patch. Two scenarios satisfy this condition. First, the break occurs before or during inflation, and inflation stretches the single biased patch across the entire observable universe. Second, the break occurs during a phase transition that is causally connected across the observable universe at the time it occurs.

The timing of the first break is therefore not a free parameter — it is a constraint imposed by the domain wall exclusion. The field equation must produce defect formation at an epoch when the observable universe is causally connected. If the equation produces defect formation only at late times when the universe is causally fragmented, the cascade mechanism fails and must be abandoned.

11.3 The Pascal Triangle Structure

The cascade follows a biased binomial distribution. A per-event bias of order 10^{-9} over a billion events produces the observed baryon-to-photon ratio. The mathematical framework exists as Pólya urn models. This structure is a geometric motivation for the cascade — the quantitative prediction requires the bias parameter to be computed from the field equation.

11.4 The Gap

Three gaps: the bias parameter from the field equation; the number of generations from cosmological dynamics; the annihilation efficiency. Additionally, the timing constraint requires the field equation to specify the epoch of defect formation relative to inflation.

11.5 Falsifiability

The baryon-to-photon ratio $\eta = (6.14 \pm 0.02) \times 10^{-10}$ must be reproduced. The discovery of a confirmed alternative mechanism would make the cascade unnecessary.

Section 12: Design Philosophy — One Equation, Two Scales

12.1 The False Divide

The mathematical physics landscape is bifurcated between continuous field theories and discrete topological models. This is not a property of nature. The FTT field equation must be a single equation whose solutions exhibit continuous behaviour in the bulk and discrete behaviour at defect sites.

12.2 Precedents

Superfluid helium-4: continuous Gross-Pitaevskii equation producing discrete quantised vortices. The closest analogy.

Solitons: the sine-Gordon equation producing discrete topological objects from continuous dynamics.

Yang-Mills instantons: continuous equations admitting sharp, countable, topologically classified objects.

12.3 The Construction Strategy

The action is S = S_EH + S_top + S_phase. S_EH is known. S_phase can be adapted from the signature-change literature. S_top is the open construction. Far from defects and phase boundaries, only the Einstein tensor survives.

12.4 What This Rules Out

Lattice regularisation as fundamental. Separate quantisation procedure. Emergent spacetime from pre-geometric structures. The ocean is continuous. The equation is already quantum because the topology is already discrete.

Section 13: Consolidation — The Research Programme

13.1 What Has Been Established

Nine requirements grounded in established mathematics or confirmed observations, each falsifiable, each mapped to existing structures, each with its gap identified. They form an interlocking system where each constrains the others. Five claims were subjected to adversarial review and revised: the dimensionality argument was re-grounded in homotopy theory; the charge-confinement picture was demoted from derivation to geometric motivation; the baryogenesis cascade was constrained by the domain wall exclusion; the phase boundary was reframed as a finite-thickness transition region; and the Cauchy problem at the Lorentzian/Euclidean interface was acknowledged as a genuine mathematical challenge.

13.2 The Three Unsolved Problems

Problem 1: The Gravitational WZW Term. The central open problem. The η-invariant and Dai-Freed theorem provide directions. Construction of S_top would unlock the entire programme.

Problem 2: RG Running. U(3) from 2T automorphisms is a finite calculation that can be attempted now.

Problem 3: Gauge Group Completion. U(1) from full rotation. SU(3) motivated by closure constraint but not derived. SU(2) from chirality mixing. All require computing symmetry groups of constrained configuration spaces.

13.3 Calculations That Can Be Done Now

Calculation 1: Minimal symplectic flux of Lorentzian metric configuration space.

Calculation 2: SU(3) from fractional twist closure constraint.

Calculation 3: Sumino protection from 2T automorphisms.

Calculation 4: The Koide angle $\delta = 2/9$ from Clifford torus geometry.

Calculation 5: Defect spectrum versus signature using the Skyrme model as proxy.

13.4 The Construction Path

Stage 1: Specification (this paper). Complete upon publication.

Stage 2: Components. Construct each action component. S_top is the open construction.

Stage 3: Assembly and verification. The work of years, possibly decades, likely requiring numerical methods.

13.5 Updated Requirements

Requirement 0 — Signature selection: stable defects only in 3+1. Grounded in homotopy theory, not fluid dynamics.

Requirement 1 — GR limit: reduces to Einstein's equations far from defects.

Requirement 2 — Topological defects: stable $\mathbb{Z}_2$ winding solutions.

Requirement 3 — Signature transition: well-defined through finite-thickness transition region; zero classical surface tension; continuous pressure gradient, not discontinuous pressure balance.

Requirement 4 — Euclidean solutions: environment-dependent minimum halo mass; Cauchy problem at interface acknowledged as open mathematical challenge.

Requirement 5 — Quantum of action: $\hbar$ from vacuum topology.

Requirement 6 — Mass hierarchy: Koide ratios from 2T symmetry.

Requirement 7 — Confinement: fractional twist as geometric motivation; SU(3) derivation outstanding.

Requirement 8 — Baryogenesis: topological cascade constrained by domain wall exclusion; first break must be pre-inflation or causally connected.

Design philosophy — One equation, two scales.

13.6 Conclusion

The spacetime field is an ocean. Matter is its patterns. Forces are interactions between patterns. Quantum numbers are topological invariants. Dark matter is a phase transition. The baryon asymmetry is memory in a topological cascade. The dimensionality of spacetime is the minimum configuration in which the ocean can knot.

These are claims. This paper has specified what the mathematics must do to substantiate them. Nine requirements, each falsifiable, each mapped to existing structures, each with its gap precisely identified. Five of these requirements were revised in response to adversarial review, and the revisions are documented.

The field equations of Field Topology Theory do not yet exist. This paper is the specification they must satisfy. If the equation can be found, it will unify gravity, quantum mechanics, and particle physics from one variational principle applied to one field. If it cannot, the specification documents exactly where the programme failed and which requirements proved incompatible.

Either outcome advances physics. A successful equation rewrites the foundations. A documented failure tells the next generation which assumptions to abandon.

The ocean is real. The question is whether we can write its equation of motion.

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